Dispersion properties of Trivelpiece-Gould waves in periodic plasma waveguides

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Spectral properties of plasma waves in periodic plasma-filled waveguides are analyzed. Unusual features in the spectral behavior and field distribution of natural plasma modes are predicted. In particular, it is shown that the plasma wave spectrum has a zonal character with allowed and forbidden bands. The periodicity strongly distorts the field distribution of plasma waves at frequencies near forbidden bands. Due to the significant contribution of higher spatial harmonics, the field of the plasma wave concentrates in small regions and achieves rather large local values. However, at frequencies far from forbidden bands the influence of the periodicity on dispersion properties and field distribution of plasma waves is negligible. Possible consequences of these effects for the beam-plasma instability in periodic plasma-filled waveguides are discussed.

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I. INTRODUCTION

Periodic waveguides have numerous applications in modern microwave technology, mainly due to their two fundamental properties: (i) support of slow waves and (ii) existence of stop bands. These properties are common for a large number of waves in periodic media regardless of their nature. However, the existence of stop bands for the plasma waves in periodic plasma-filled waveguides is proved here in a rigorous manner.

Recently, periodic waveguide structures loaded with a magnetized plasma have become a subject of intense experimental and theoretical investigation [1–4]. Since they provide extremely favorable conditions for beam-wave instability development, they can be useful for enhancement of power-handling capability, increase of tunability, and improvement of some other characteristics of high power microwave (HPM) electron devices.

At present, there are several types of comparatively well-developed plasma HPM sources [5–7] which are capable of outperforming their vacuum counterparts in many technical aspects. The influence of plasma on the operation of some other HPM sources has been studed also in [8–11]. However, the area is open for in-depth research to understand the physics of the plasma influence. Despite numerous experimental observations of plasma-induced improvement of HPM sources, the mechanism of the plasma effect on their operation still remains unclear. To a great extent this can be explained by lack of understanding of the spectral properties of plasma-filled periodic structures at low frequencies ($\omega < \omega_p$, where ω_p is the plasma frequency and ω is the wave frequency).

As known, plasma-filled periodic waveguides support two families of modes [12,13]. One of them is the high-frequency electromagnetic (EM) mode. Dispersion properties of EM modes in plasma-filled periodic waveguides are very similar to those in vacuum periodic waveguides. They have been successfully analyzed by using conventional approaches [12,13] and the results confirmed by experimental data [7,14,15]. Two main features inherent to EM modes in vacuum periodic waveguides, the appearance of stop bands and slow waves, are shown to remain qualitatively the same for EM modes in plasma-filled periodic waveguides also.

The other family is the purely plasma modes, which are counterparts of the well-known Trivelpiece-Gould (TG) modes in smooth plasma waveguides [16]. They have been observed in experiments [7,17]; however, in-depth analysis of their spectral properties is extremely difficult in both theory and experiment. Earlier theoretical considerations of the TG modes in periodic waveguides were also based on the conventional approaches [12,18], like expanding the fields in spatial harmonic series and then applying boundary conditions on periodic walls. The dispersion relation obtained in the form of an infinite determinantal equation was calculated by truncating it to finite size. It has been shown by many workers [12,13,18,19] that periodicity acts on the dispersion properties of TG modes approximately in the same manner as on those of EM modes, namely, the dispersion diagram for the periodic waveguide TG modes consists of a set of shifted (in periodicity) dispersion curves for TG modes in a smooth waveguide. They undergo splitting at the points of mutual crossing. Near these points stop bands are formed if the group velocities of the modes corresponding to the crossing curves have opposite signs. However, later, more detailed investigations showed that the dispersion diagrams calculated in such a manner, if a large number of spatial harmonics were taken into account, lost the periodicity with respect to the wave number required by the Floquet theorem [20]. New passbands and forbidden bands appear and disappear when every subsequent spatial harmonic is taken into account. It seems that it is impossible to obtain correct results regardless of the number of spatial harmonics. Meanwhile, the density of dispersion curves increases infinitely in the finite range of frequency $0 < \omega < \omega_p$ producing a fundamentally different type of spectral behavior, the so-called dense spectrum [21]. The properties of the latter have not been studied yet even qualitatively.

Recently, a more suitable approach for the treatment of the ‘‘dense’’ spectrum was suggested [22]. It can take into account all spatial harmonics and the corresponding eigenvalue/eigenfunction problem can be reformulated in

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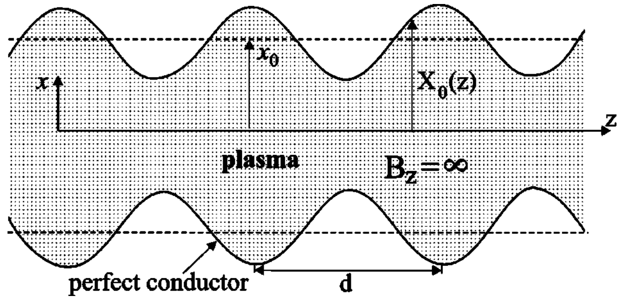

FIG. 1. Geometry of the problem.

terms of an integral equation for the total field, avoiding the infinite determinantal dispersion equation. The integral equation obtained was solved analytically in the case of $|\varepsilon(\omega)| \ll 1$, where $\varepsilon(\omega)=1-\omega_p^2/\omega^2$ is the dielectric permittivity of the cold plasma. The dispersion curves obtained in the first approximation had no stop bands, but it was unclear whether this was the consequence of the idealization or if such a feature is really inherent in the TG modes in periodic waveguides.

The aim of this work is to develop a more accurate and detailed analysis of the dispersion properties of the TG modes in a periodic plasma-filled waveguide based on the approach proposed in [22]. The remainder of the article is organized as follows. Section II deals with the formulation of the problem and the derivation of a basic functional equation suitable for numerical analysis. Section III presents several numerical schemes to solve the obtained functional equation. Section IV contains the computed results. Section V summarizes the findings of this work.

II. FORMULATION OF THE PROBLEM AND DESCRIPTION OF THE MATHEMATICAL MODEL

For the sake of simplicity consider, following [22], a planar waveguide of periodically varying thickness loaded with a uniform, cold, collisionless plasma embedded in a strong axial static magnetic field (see Fig. 1). All wave perturbations are assumed to be of the TM polarization $(E_x, H_y, E_z) \sim e^{-i\omega t}(\omega < \omega_p)$, electrostatic, and symmetric with respect to the z axis $[E_z(x,z)=E_z(-x,z)]$. Dispersion properties of the TG modes in such a structure are characterized by the functional equation [22]

$$e^{ik_z\varphi(z)}\Psi(z+\varphi(z))[1+\varphi'(z)]+e^{-ik_z\varphi(z)}\Psi(z-\varphi(z))\times[1-\varphi'(z)]=0, \tag{1}$$

where $\Psi(z)=e^{-ik_z z}E_z(0,z)$, $\varphi(z)=|\varepsilon(\omega)|^{1/2}X_0(z)$, $x=X_0(z)$ is the equation of the periodic waveguide boundary, $X_0(z+d)=X_0(z)$, d is the period of the structure, k_z is the wave number of the perturbations, and $E_z(0,z)=E_z(x,z)|_{x=0}$ is the axial electric field on the waveguide axis, which is assumed to be quasiperiodic, $E_z(0,z+d)=e^{ik_z d}E_z(0,z)$. The prime denotes differentiation. Other components of the field can be expressed through $E_z(x,z)$ as

$$\frac{\partial H_y}{\partial x}=-ik\varepsilon(\omega)E_z, \quad E_x=\frac{1}{ik}\frac{\partial H_y}{\partial z}. \tag{2}$$

Equation (1) is correct in a rigorous mathematical sense provided that $\varphi'(z)\leqslant 1$ and valid in a qualitative sense if this condition is slightly broken [23].

A direct numerical analysis of Eq. (1) is hardly possible, since it describes the dense spectrum, i.e., each point in the ω-k_z plane inside the frequency range $0<\omega<\omega_p$ is a solution of Eq. (1) or is infinitely close to it. Indeed, let $k_{zm}(\omega)$ be the eigenvalue of Eq. (1) with the eigenfunction $\Psi_m(z)$, where m is the transverse index. It is not difficult to see that $k_{zmn}(\omega)=k_{zm}(\omega)+nk_0$, where $k_0=2\pi/d$, are also eigenvalues of Eq. (1). But they have other eigenfunctions: $\Psi_{mn}(z)=\Psi_m(z)e^{-ink_0 z}$. The set of curves $k_{zmn}(\omega)=k_{zm}(\omega)+nk_0$, where $m=1,\ldots,\infty$ and $n=0,\pm 1,\pm 2,\ldots,\pm\infty$, entirely fill the range $0<\omega<\omega_p$ in the ω-k_z plane [21]. However, it can be easily established that eigenvalues $k_{zmn}(\omega)$ and eigenfunctions $\Psi_{mn}(z)$ with fixed m and different n define the same full field $E_{zm}(0,z)$, which is characterized by the transverse index m. Therefore, one can use any of them to calculate the full field (the others are spurious).

At a fixed n the eigenvalues of Eq. (1), $k_m(\omega)$, correspond to a set of transverse modes and consequently generate the ordinary spectrum as in the case of a smooth plasma-filled waveguide. Thus, before numerical calculations it would be reasonable to transform Eq. (1) in such a way that one can get rid of spurious solutions.

For this purpose, we introduce a function

$$F(z)=\int^z E_z(0,z')dz'.$$

For $F(z)$ we have the following problem:

$$F(z+\varphi(z))+F(z-\varphi(z))=0,$$
$$F(z+d)=e^{ik_z d}F(z). \tag{3}$$

Now present $F(z)$ in the form $F(z)=\rho(z)e^{ik_z z+i\theta(z)}$, and decouple Eq. (3) into the folowing two problems for two new unknown real functions $\rho(z)$ and $\theta(z)$:

$$\rho(z+\varphi(z))=\rho(z-\varphi(z)),$$
$$\rho(z+d)=\rho(z), \tag{4}$$

$$\frac{1}{2}[\theta(z+\varphi(z))-\theta(z-\varphi(z))]=(m+1/2)\pi-k_z\varphi(z),$$
$$\theta(z+d)=2\pi n+\theta(z). \tag{5}$$

It can be shown that the spectral properties of our structure are determined by the problem (5), while the problem (4) has a simple and evident solution $\rho(z)=\text{const}$.

Now, assume $n=0$ in Eq. (5); this enables us to get rid of spurious solutions. The resulting problem characterizes the ordinary spectrum of transverse modes:

$$\frac{1}{2}[\theta(z+\varphi(z))-\theta(z-\varphi(z))]=(m+1/2)\pi-k_z\varphi(z),$$
$$\theta(z+d)=\theta(z). \tag{6}$$

Therefore it is possible to analyze Eq. (6) numerically.

III. METHODS OF SOLUTION OF BASIC EQUATION

The simplest numerical solution of Eq. (6) is based on perturbation theory. The zero approximation can be found by expanding $\theta(z\pm\varphi(z))$ into the Taylor series near z:

$$\frac{1}{2}[\theta(z+\varphi(z))-\theta(z-\varphi(z))]$$
$$=\theta'(z)\varphi(z)+\frac{1}{6}\theta'''(z)\varphi^3(z)+\cdots. \tag{7}$$

Taking in to account only the first term of the Taylor series in Eq. (7) and then substituting Eq. (7) in Eq. (6), one can obtain a first-order differential equation for $\theta(z)$ that can be solved analytically:

$$\theta(z)=-k_z z+(m+1/2)\pi\int^z\frac{dz'}{\varphi(z')}. \tag{8}$$

Using the periodicity of $\theta(z)$, it is not difficult to find expressions for the spectral curves (i.e., eigenvalues as functions of ω):

$$k_{zm}(\omega)=\frac{(m+1/2)\pi}{d}\int_0^d\frac{dz'}{\varphi(z')}. \tag{9}$$

For the eigenfunctions we have [assuming $\rho(z)=1$]

$$F_m(z)=\exp\left(i(m+1/2)\pi\int^z\frac{dz'}{\varphi(z')}\right). \tag{10}$$

Expressions (9) and (10) have clear physical meaning. Recall that $k^l_{zm}(z)=(m+1/2)\pi/\varphi(z)$ is the local wave number of a smooth plasma waveguide with local thickness $X_0(z)$. Then it can be concluded that the eigenfunctions have the form of wave packets $F_m(z)=\exp[i\int^z k^l_{zm}(z')dz']$, which are typical for slightly nonuniform plasma configurations [24] and widely accepted for description of slightly nonuniform waveguides [25]. Thus, expression (9) is the local wave number of the smooth waveguide averaged over the period. In the case of sinusoidal ripples $X_0(z)=x_0(1+\alpha\cos k_0 z)$, where x_0 is the mean thickness of the waveguide, the integration in Eq. (9) can be carried out analytically, yielding

$$k_{zm}(\omega)=\frac{(m+1/2)\pi}{|\varepsilon(\omega)|^{1/2}x_0\sqrt{1-\alpha^2}}. \tag{11}$$

In the limit of $\alpha\to 0$ expressions (10) and (11) tend to eigenfunctions and eigenvalues corresponding to the ordinary Trivelpiece-Gould waves in a smooth waveguide:

$$F_m(z)=\exp[ik_{zm}(\omega)z],\quad k_{zm}(\omega)=\frac{(m+1/2)\pi}{|\varepsilon(\omega)|^{1/2}x_0}.$$

More accurate solutions of Eq. (5) can be found using perturbation theory. Rewrite Eq. (5) in the operator form $\hat{L}\theta(z)=f(z)$, where

$$\hat{L}\theta(z)\equiv\frac{1}{2}[\theta(z+\varphi(z))-\theta(z-\varphi(z))]$$

and

$$f(z)=(m+1/2)\pi-k_z\varphi(z).$$

Presenting $\hat{L}$ in the form $\hat{L}=\hat{L}_0+\hat{L}_1$ and $\theta(z)$ as $\theta(z)=\theta_0(z)+\theta_1(z)+\cdots$, where $\hat{L}_0=\varphi(z)(\partial/\partial z)[\cdots]$, $\hat{L}_1=\hat{L}-\hat{L}_0$, we have

$$(\hat{L}_0+\hat{L}_1)[\theta_0(z)+\theta_1(z)+\cdots]=f(z). \tag{12}$$

Assuming that $\hat{L}_0$ and $\theta_0(z)$ are the zero-order values, $\hat{L}_1$ and $\theta_1(z)$ are the first-order values, and so on, we obtain the following recurrence chain: $\hat{L}_0\theta_0(z)=f(z)$, $\hat{L}_0\theta_1(z)=-\hat{L}_1\theta_0(z)$, ..., $\hat{L}_0\theta_n(z)=-\hat{L}_1\theta_{n-1}(z)$. This is a chain of first-order ordinary inhomogeneous differential equations. Each of them has an analytical solution. Every successive term of the chain can be expressed through the preceding one in a simple analytical way. Omitting algebraic details, we represent the expressions for eigenvalues and eigenfunctions obtained after N iterations as

$$k^N_{zm}(\omega)=\frac{(m+1/2)\pi}{d}\int_0^d\frac{dz'}{\varphi(z')}\left(1+\sum_{n=1}^N S_n(z')\right), \tag{13}$$

$$F^N_m(z)=\exp\left[i(m+1/2)\pi\int^z\frac{dz'}{\varphi(z')}\left(1+\sum_{n=1}^N S_n(z')\right)\right], \tag{14}$$

where $S_n(z)$ are defined by the following recurrence rule:

$$S_1(z)=1-\tfrac{1}{2}\int_{z-\varphi(z)}^{z+\varphi(z)}dz'/\varphi(z'),$$

$$S_2(z)=S_1(z)-\tfrac{1}{2}\int_{z-\varphi(z)}^{z+\varphi(z)}dz'S_1(z')/\varphi(z'),\ \ldots,$$

$$S_n(z)=S_{n-1}(z)-\tfrac{1}{2}\int_{z-\varphi(z)}^{z+\varphi(z)}dz'S_{n-1}(z')/\varphi(z').$$

As follows from Eq. (7), convergence of solutions (13) and (14) should be expected when $\varphi(z)<(\sqrt{6}/2\pi)d$, which is confirmed by computations. If this inequality is broken, the series in Eq. (13) and (14) have a poor convergence or divergence. Moreover, no periodic solutions exist for the fre

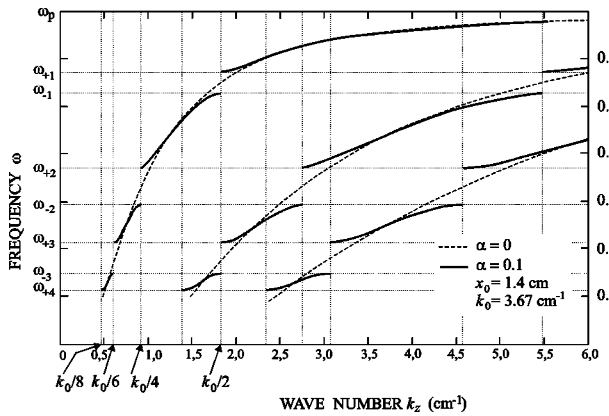

FIG. 2. Dispersion curves for the first three plasma modes.

quencies providing $\varphi(z)=ld/2$ at some $z=z_0$, where $l=1,2,\ldots,\infty$. This point is a singular one for the operator $\hat{L}$: $\hat{L}\theta(z)|_{z=z_0}=0$. Inside the frequency bands where it is valid, no propagating modes can occur. So the former can be treated as stop bands. It is possible to obtain analytical expressions for them: $\omega_{-l}<\omega<\omega_{+l}$, where $\omega_{\pm l}=\omega_p\{1+[\pi l/x_0k_0(1\pm\alpha)]^2\}^{-1/2}$.

Near stop bands it is reasonable to solve Eq. (6) by direct numerical methods. Two of them have been developed. The first is based on the expansion of $\theta(z)$ in terms of a Fourier series: $\theta(z)=\Sigma_{q=-\infty}^{\infty}C_q e^{ik_0qz}$. The resulting infinite system of linear algebraic equations was solved after truncation. Usually, about 50 Fourier harmonics ($|q|\leqslant 50$) were found enough to achieve precision within 10^{-4} for the calculation of dispersion curves including regions near stop bands.

Another approach was based on the approximation of $\theta(z)$ by spline functions:

$$\theta(z)\cong\sum_{i=0}^{n-1}\theta_i B_i^k(z), \tag{15}$$

where

$$B_i^0=\begin{cases}1 & \text{if} \quad z_i\leqslant z<z_{i+1}\\ 0 & \text{otherwise,}\end{cases}$$

$$B_i^k(z)=\left(\frac{z-z_i}{z_{i+k}-z_i}\right)B_i^{k-1}(z)+\left(\frac{z_{i+k+1}-z}{z_{i+k+1}-z_{i+1}}\right)B_{i+1}^{k-1}(z),$$

$$k\geqslant 1,$$

$$z_i=\frac{i}{n}d, \quad i=0,1,\ldots,n-1.$$

To achieve accuracy within 10^{-4} in the calculations of dispersion curves, one needs to take $n\sim 100$ for the first-order spline functions. By increasing the number of Fourier harmonics or the number of spline functions, it is possible to calculate the dispersion curves and field distribution with any desired accuracy.

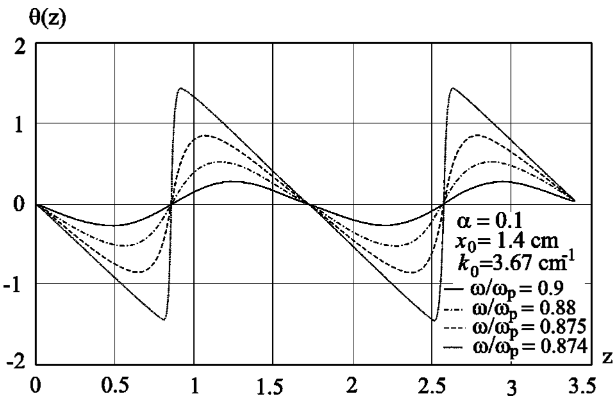

FIG. 3. Phase shift of the field $\theta(z)$ caused by the periodicity taken at different frequencies within the first passband $\omega_{+1}<\omega<\omega_p$.

IV. ANALYSIS OF NUMERICAL RESULTS

Figure 2 shows the dispersion curves of the three first plasma modes ($m=0,1,2$) for a sinusoidally rippled waveguide. The dispersion properties and field distribution of plasma waves are modified slightly by the periodicity compared to those of the smooth waveguide if the frequency is far from a stop band, but they change significantly when approaching the stop bands. Group velocities of plasma waves at the boundaries of the stop bands ($\omega=\omega_{\pm l}$) tend to zero. It should be specially noted that the wave number of the fundamental plasma mode tends to $k_0/2l$ if the frequency approaches the lth stop band. No stop bands occur at $\omega>\omega_{+1}$. Therefore, the results of the approximate analysis developed in [22] are valid only in this region. In the region of low frequencies the stop bands start overlaping. For example, for the parameters considered, $\omega_{+6}>\omega_{-5}$, so no propagating waves occur below $\omega_{+5}\cong 0.339\omega_p$. However, the rigorous mathematical validity of our initial equation (1) $[\alpha x_0k_0|\varepsilon(\omega)|^{1/2}\leqslant 1]$ [23] is restricted to the range $\omega\geqslant 0.457\omega_p$ for the case in hand. Thus, in this region the condition of validity of our consideration is slightly violated

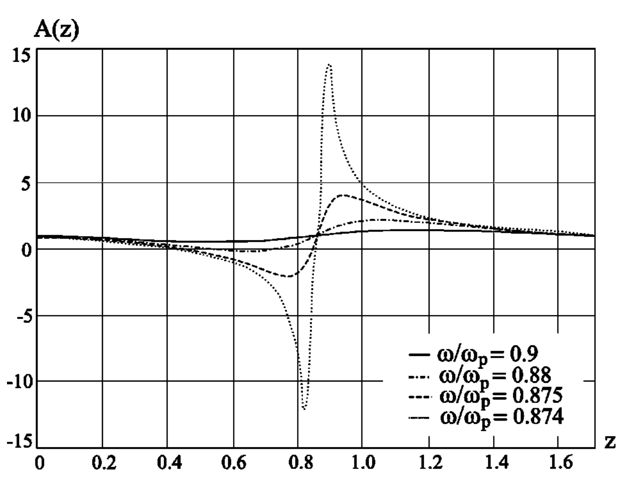

FIG. 4. Variation of the field amplitude $A(z)$ caused by the periodicity taken at different frequencies within the first passband $\omega_{+1}<\omega<\omega_p$.

and the results should be considered as qualitative.

To evaluate the influence of the periodicity on the field distribution, we represent the field in the form $E_z(0,z) = A(z)e^{ik_z z + i\theta(z)}$. For a smooth waveguide one can put $A(z)=1$ and $\theta(z)=0$. Figures 3 and 4 show the phase shift $\theta(z)$ and variable amplitude $A(z)$ of the electric field $E_z(0,z)$ at different frequencies inside the first passband $\omega_{+1}<\omega<\omega_p$. The behavior of $\theta(z)$ is close to sinusoidal if the frequency is far from ω_{+1}. However, it changes sharply if the frequency approaches $\omega_{+1}\cong 0.874\omega_p$. Near ω_{+1}, $\theta(z)$ has a profile close to a serrated one while $A(z)$ has very sharp maxima. Such significant modifications are caused by the contributions of a large number of higher spatial harmonics whose amplitudes drop very slowly with increasing number if the frequency is near a stop band. This leads to the concentration of the field in small regions where it can achieve a large magnitude. Such behavior is also observed when the frequency approaches any boundary of a stop band $\omega=\omega_{\pm l}$. Note that this is hardly possible in vacuum periodic waveguides where stop bands are formed by interaction of two or very rarely more spatial harmonics.

V. CONCLUSION

Thus, the results obtained on the basis of the accurate analysis show that the TG waves in periodic plasma waveguides have certain specific features as compared to the EM waves in periodic waveguides. First, there are the frequency bands where the periodicity slightly modifies the dispersion properties of the plasma waves. They propagate there in the form of wave packets (14) close to those usually appearing in a slightly inhomogeneous plasma [24] and typical for slightly inhomogeneous waveguides [25].

Second, there are the frequency bands where the periodicity leads to significant modifications as compared to the smooth plasma-filled waveguides both in dispersion properties of the plasma waves and in the field distributions. As in the case of the EM modes, the formation of stop bands is possible. These effects can hardly be described by perturbation theory [see representations (13) and (14)]. Two schemes of direct numerical analysis have been developed to solve the basic functional equation (6). They are based on the Fourier expansion and spline function approximation, respectively. They show that the quantity and widths of stop bands depend on the parameters of the periodic plasma waveguide. The widths of stop bands are proportional to the plasma frequency and increase with increase in the height of ripples at a fixed period. They also increase with decrease of the structure period at a fixed height of ripples. When the frequency of some plasma mode approaches the stop band, the field concentrates in small regions (much smaller than the structure period), where it can achieve a value exceeding its averaged value by a factor 10. Meanwhile, the group velocities of plasma modes tend to zero. Such effects can be explained by the significant influence of higher spatial harmonics.

The revealed features of the TG modes in periodic plasma waveguides are essential for the treatment of the beam-plasma instability in periodic plasma-filled waveguides, as the local field distribution of plasma waves can strongly modify the equilibrium plasma density and the beam bunching process responsible for the appearance of coherent radiation. Also, the stop bands for plasma waves have been predicted and can be observed experimentally to confirm the validity of the analysis presented here.

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